

A CONSTRUCTIVE METHOD FOR SOLVING A MULTI-OBJECTIVE LINEAR PROGRAM WITH BOUNDED VARIABLES

Mohamed Boualem^{*1}, Nassim Touche^{†‡2}, and Djamil Aïssani^{§3}

^{1,3}Research Unit LaMOS, Faculty of Technology, University of Bejaia, Algeria

²Research Unit LaMOS, Operational Research Department, University of Bejaia, Algeria

ABSTRACT

The search for an optimal solution remains a central objective in optimization, but in many situations the optimal solution cannot be obtained in polynomial time. For this, it would sometimes be necessary to find a good solution close to the optimum that can be obtained in a reasonable time. The aim of this paper is to find an ε -efficient or efficient solution for a multi-objective linear programming problem with polyhedral constraints and bounded decision variables. For this, we define an ε -efficient solution for which a characterization theorem is proved, and where the ε -optimality criterion is formulated simultaneously for all the objectives of the problem. By using this concept of ε -efficiency, we develop an algorithm that combines the well known Benson's Procedure in multi-objective linear programming and the Adaptive Method elaborated for finding ε -optimal solutions in mono-objective linear programming, based on the diminution of the suboptimality estimate. The proposed algorithm is illustrated by a numerical example and finally, it is implemented in Matlab to solve a set of generated test problems. The efficient solutions found are located on the Pareto fronts of the same test problems treated by using the well-known Gamultiobj algorithm of Matlab Optimization Tool.

KEYWORDS: Multi-Objective Linear Programming; Subefficient Solutions; Benson's Procedure; Adaptive Method.

1 Introduction

This paper deals with a multi-objective linear programming problem with bounded variables. This class of problems refers to optimization problems that have several conflicting objectives, and where all objectives and constraints are linear and the decision variables are bounded. It is well known that the optimality conditions in this case differ from the mono-objective case, in the sense that optimality principles based on dominance are used to analyze multi-objective problems. So, to get a good compromise solution, it is necessary to define dominance relations between the different criteria.

Practical and theoretical interest in solving multi-objective linear problems has attracted attention of many researchers, and several methods are proposed for their numerical resolution. In the literature, we can distinguish two classes of these methods which are scalarization and non-scalarization methods (we refer the reader to [6] for more details). In both cases different techniques for solving linear multi-objective problems are used such as Methods based on the well-known simplex method in [7] and the interior-point algorithm introduced by [10] for the mono-objective linear programming,

^{*}mohamed.boualem@univ-bejaia.dz

[†]nassim.touche@univ-bejaia.dz

[‡]Corresponding Author

[§]djamil.aissani@univ-bejaia.dz

extended to the multi-objective case in [2] and [4]. Recently, other approaches have been proposed such as Robust optimization to solve a scalarized multi-objective linear program in [5], the multiple reduced gradient method for multi-objective optimization problems in [9] and the generalization of the reduced gradient method in [8].

Notation The set $\mathbb{R}_+^p = \{\lambda \in \mathbb{R}^p : \lambda > 0\}$ denotes the positive orthant of \mathbb{R}^p .

2 Problem Statement and Definitions

Consider the following Multi-Objective Linear Programming (MOLP) problem with bounded variables:

$$\max \quad Cx = (C_1^T x, C_2^T x, \dots, C_p^T x)^T, \quad (1)$$

$$Ax = b, \quad (2)$$

$$l \leq x \leq u, \quad (3)$$

where A is an $m \times n$ -matrix with $\text{rank}(A) = m < n$; b is an m -vector; x, l, u are n -vectors; C is a $p \times n$ -matrix whose rows are n -vectors $C_k^T, k = 1, \dots, p$. The symbol $(^T)$ represents the transposition operation.

Definition 2.1. The couple $\{x, J_B\}$ formed by the feasible solution x and the support J_B is called a *Support Feasible Solution (SFS)* of the problem (1)-(3). The SFS $\{x, J_B\}$ is said to be *nondegenerate*, if $l_j < x_j < u_j, \forall j \in J_B$.

3 Efficient Solutions of the MOLP Problem

In the absence of an optimality criterion that maximizes the objective functions simultaneously in the multi-objective problem (1)-(3), it is necessary to identify the best compromise solution by defining order dominance relations between these objectives. That yields the so-called efficient solutions (or Pareto optimal solutions) on which we develop the solution method.

Remark 3.1. The support feasible solution is a more general concept than the basic one. A support feasible solution can be an interior point, a boundary point or an extreme point of X , while a basic feasible solution is always an extreme point.

4 Tables and figures

Value 1	Value 2	Value 3
α	β	γ
1	1110.1	a
2	10.1	b
3	9.31	c
4	5.23	c
5	2.32	c
6	10.1	b
7	9.31	c
8	5.23	c

Table 1: Table with aligned units.



Figure 1: Cap Carbon dans le Parc National de Gouraya

5 Conclusion

In this paper, we carried out a theoretical study and used a constructive approach for finding a numerical solution of a multi-objective linear programming problem with bounded variables. The aim of this study is to construct an algorithm to find a sub-efficient solution in the considered problem. For this, we presented the efficient solutions, then characterized the ε -efficient solutions for a multi-objective linear programming problem with bounded variables. Based on a characterization theorem that we have proven and formulated for all objectives simultaneously, and combined with the Benson's Procedure, we have developed an algorithm that allows us to find an ε -efficient solution. We have presented a numerical example that shows the quality of the obtained solution and an improvement in every iteration can be guaranteed by means of the suboptimality estimate. Finally, we tried on a set of generated test problems, to situate the solutions obtained by the proposed algorithm, with respect to the Pareto fronts obtained by the well-known Gamultiobj algorithm of Matlab Optimization Tool. We can see from the results that the solutions obtained by the proposed method are good quality solutions, and located on the Pareto fronts of the problems treated. In a future work, we will try to develop an algorithm to determine all ε -efficient solutions for the considered problem and implement the method in order to make a comparison with the benchmark problems in the literature.

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